Towards dynamic catchment modelling: a Bayesian Hierarchical mixtures of experts framework

Marshall et al.(2007)

이종진

Seoul National University

ga0408@snu.ac.kr

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- There is not a catchment model that will perform consitently over the wide range of conditions that exist
  - A modeller must choose the most appropriate model ($\rightarrow$ modeller's preference, familiarity with particular models)
- Because of model uncertainty,
  - Combine the results from several different hydrogical models.(like Bayesian Model Averaging)
- Hierarchical Mixtures of Experts(HME) is a method of combining model results which allows the individual model weights to be estimated dynamically

HME Framework

- HME models have probabilistic switch between model structure.
- This switch is dependent on the current hydrological 'state' of the catchment.(by user-defined predictor variables.)
- It consists of
  - Expert Network(Component models)
  - Gating Network(weights, ex: logisitic function)

Two-level HME

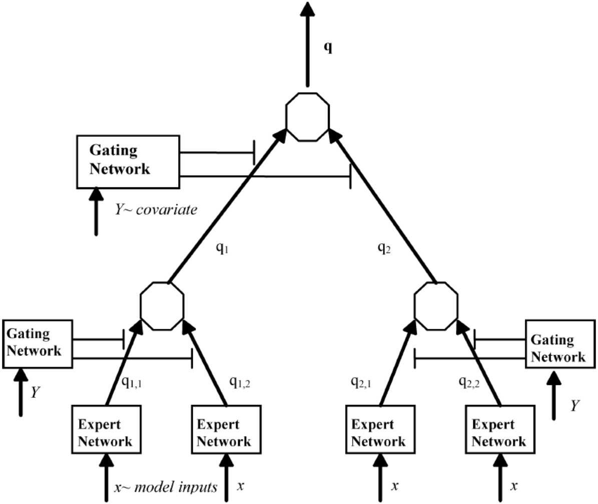

Figure: Two-level HME

HME Framework

- User-defined predictor variables which contain conditions of the current state of the catchment are used.
- Expert Network
  - Data is divided probabilistically based on some exogenous factors.
  - Models are fitted to the data that fall in each region.
- Gating Network.
  - Mathmatical function that assigns a probability to each model based on the predefined predictor.
  - For two-component models, a simple choice for the gating function,

$$g_{t,1} = \frac{e^{\beta Y_t}}{1 + e^{\beta Y_t}}$$

  - Monte Carlo (MCMC) methods are used to estimate gating and individual model parameters.

Case study

- Modeling rainfall-runoff model.
- 10 Catchments are chosen to vary in terms of size, location, and yield in Australia,
- One level, two component HME is applied with simplified Australian Water Balance Model.

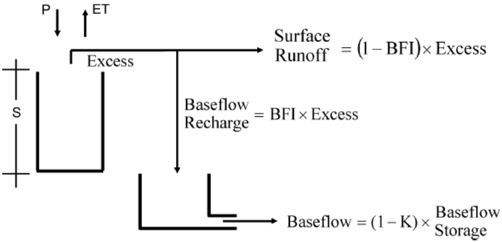

Figure: AWBM

Case study

- AWBM parameters K, BFI and S
  - Highly interdependent $\rightarrow$ adaptive Metropolis algorithm.
  - However, the algorithm is applied when sampling all model parameters as a whole and may not be suited to HME structure.
  - Instead, pre-tuning runs were performed with the single AWBM using the adaptive Metropolis alogorithm.
  - 50,000 iterations for each catchments, and this yields an estimate of the covariance of parameters' posterior distributions.
  - Then, Metropolis algorithm was used to sample the AWBM parameters.
  - Multivariate normal proposal distribution with mean at the current value and fixed covariance based on the pre-tuning run of the Metropolis algorithm

Case study

- Gating network parameters β
  - Linear logistic function.
  - Metropolis algorithm.
  - Multivariate normal proposal distribution with mean at the current value and fixed covariance
- Variance σ^2
  - Proposal distribution

$$\sigma^{2'} \sim \chi^{-2}\{\nu = 4 + 2(\sigma^2)^2/\sigma_\theta^2, \lambda = 2\sigma^2[1 + (\sigma^2)^2]$$

  - $\sigma^{2'}$ is the proposed value, σ^2 is the current value, σ_θ^2 is proposeal variance that is tuned to get proper acceptance rate

Case study

- Prior densities.
  - Diffuse or vague priors are used.
  - For gating function coefficients, prior are normal with mean zero and large variances.
- Likelihood function
  - Applying a Box-Cox transformation, $\rightarrow$ Uncorrelated error term.

$$p(Q|\theta) = (2\pi\sigma^2)^{-n/2} \times \prod \exp(-\{\log[(Q_t + \lambda_2)/f(x_t;\theta) + \lambda_2)]\}_t^2/2\sigma^2) \times (Q_t + \lambda_2)^{-1}$$

  - $p(Q|\theta)$ is the likelihood, Q_t is the observed streamflow at time step t, $f(x_t;\theta)$ is the calculated flow at time step t, n is the length of the data, x_t is the set of inputs at time t, θ is the set of model parameters, andλ is set at 0.5

Case study

Catchment	AWBM		Two-component HME Nash–Sutcliffe coefficient	HME maximum log-likelihood
	Nash–Sutcliffe Coefficient	Maximum log-likelihood		
A	0·74	−495	0·88	4183
B	0·58	1345	0·80	22 086
C	0·81	−2273	0·92	−725
D	0·41	−4677	0·75	6266
E	0·51	9278	0·87	15 432
F	0·66	−5264	0·86	81
G	0·52	19 942	0·58	22 134
H	0·76	−3033	0·91	−183
I	0·81	−14 882	0·92	−9527
J	0·79	−3752	0·91	−2259