

# Individual Fairness AI Reviews

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August 4, 2020

# Table of Contents

## Metric Based

Fairness Through Awareness (2012)

Probably Approximately Metric-Fair Learning (2018)

## Bandit Problem Based

Online Learning with an Unknown Fairness Metric (2018)

## Counterfactual Based

Counterfactual Fairness (2017)

When Worlds Collide: Integrating Different Counterfactual Assumptions in Fairness (2017)

## Multiple classifications based

Average Individual Fairness (2019)

# Table of Contents

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Fairness Through Awareness (2012)

Probably Approximately Metric-Fair Learning (2018)

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Online Learning with an Unknown Fairness Metric (2018)

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Counterfactual Fairness (2017)

When Worlds Collide: Integrating Different Counterfactual Assumptions in  
Fairness (2017)

## Multiple classifications based

Average Individual Fairness (2019)

## Introduction

- ▶ An algorithm is **individual fair** if it gives similar predictions to similar individuals, i.e.,

$$\left| P(\hat{Y}_i = y | X_i) - P(\hat{Y}_j = y | X_j) \right| \leq \epsilon; \text{ if } d(X_i, X_j) \approx 0$$

where  $i, j$  denote two individuals.

- ▶  $d(i, j)$  is a distance metric between two individuals, and here we assume  $d(i, j)$  is given for specific task.

## Treating similar individuals similarly

- ▶ In a binary classification problem, we consider randomized mappings  $M: \mathcal{X} \rightarrow \Delta(\mathcal{Y})$  from individuals to probability distribution over outcomes. To classify  $x \in \mathcal{X}$  choose an outcome  $y$  according to the distribution  $M(Y = y|x)$ .
- ▶ Find a mapping from individuals to distribution over outcomes that minimizes expected loss **subject to the  $(D, d)$ -Lipschitz condition**,

$$D(M(\cdot|x), M(\cdot|x')) \leq d(x, x'), \quad \forall x, x' \in \mathcal{X}$$

where  $D$  is a measure of similarity of distributions.

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where  $D$  is a measure of similarity of distributions.

## Treating similar individuals similarly

- ▶ Denote  $\mathcal{I}$  as an instance, and  $L$  as a loss function.
- ▶ The Fairness LP:

$$\begin{aligned} \text{opt}(\mathcal{I}) &\stackrel{\text{def}}{=} \min_M \int_x \sum_{y \in \{0,1\}} \sum_{\hat{y} \in \{0,1\}} L(y, \hat{y}) M(Y = \hat{y} | x) P(x, Y = y) dx \\ &\text{subject to } \forall x, x' \in \mathcal{X} : D(M(\cdot | x), M(\cdot | x')) \leq d(x, x') \end{aligned} \quad (1)$$

## Probability metrics

- ▶ Let  $P, Q$  denote probability measures on a finite domain  $A$ .
- ▶ Total variation norm between  $P$  and  $Q$ :

$$D_{\text{tv}}(P, Q) = \frac{1}{2} \sum_{a \in A} |P(a) - Q(a)|$$

- ▶ Relative  $\ell_\infty$  norm between  $P$  and  $Q$ :

$$D_\infty(P, Q) = \sup_{a \in A} \log \left( \max \left\{ \frac{P(a)}{Q(a)}, \frac{Q(a)}{P(a)} \right\} \right)$$

- ▶ **Lemma.** Let  $D \in \{D_{\text{tv}}, D_\infty\}$ . Given an instance  $\mathcal{I}$ , we can compute  $\text{opt}(\mathcal{I})$  with a linear program of size  $\text{poly}(|\mathcal{X}|, |\mathcal{Y}|)$  (w.r.t.  $M$ ).



# Table of Contents

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Online Learning with an Unknown Fairness Metric (2018)

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## Introduction

- ▶ In the previous work, the individual-fair learning algorithm is computationally intractable (even for simple fair-learning tasks).
- ▶ Suppose a similarity metric  $d$  is given.
- ▶ The author proposed an approximately individual-fair condition which is a relaxed version of the previous individual fairness.

## Approximate Metric-Fairness

- ▶ **Def.** A predictor  $h$  is  $(\alpha, \gamma)$ -approximately metric-fair (MF) w.r.t. a similarity metric  $d$  and a data distribution  $\mathcal{D}$  if

$$\mathcal{L}_\gamma^F := P_{x, x' \sim \mathcal{D}}[|h(x) - h(x')| > d(x, x') + \gamma] \leq \alpha.$$

- ▶ If  $\alpha = 0$ , then  $\mathcal{L}_\gamma^F$  means perfect MF.
- ▶ Notation
  - ▶  $H^{\alpha, \gamma}$ : the set of functions which satisfy  $(\alpha, \gamma)$ -approximate MF on  $\mathcal{D}$
  - ▶  $\widehat{H}^{\alpha, \gamma}$ : the set of functions which satisfy  $(\alpha, \gamma)$ -approximate MF on the training set.

## Fair learning

- ▶ Objective for fair learning:

$$\text{minimize}_h \text{err}_S(h) \text{ subject to } h \in \widehat{H}^{\alpha, \gamma}$$

where  $S$  is a training set,  $\text{err}_S(h)$  denotes the expected  $\ell_1$  error of  $h$ .

- ▶ Since  $\widehat{H}^{\alpha, \gamma}$  in the constraint induces 0/1 loss, they use empirical  $\ell_1$  MF violation  $\xi_S(h)$  given by

$$\xi_S(h) = \sum_{x, x' \in S} \max(0, |h(x) - h(x')| - d(x, x')).$$

- ▶ For some  $\tau \in [0, 1]$ ,

$$\text{minimize}_h \text{err}_S(h) \text{ subject to } \xi_S(h) \leq \tau.$$

## Main contributions

- ▶ (Generalization) This fair learning guaranteeing fairness not just for the training sets but also for the underlying population distribution, under some conditions.
- ▶ (Efficiency) This algorithm guarantees to construct polynomial-time learning algorithm which satisfies approximate MF and best-possible accuracy (for classes of linear and logistic predictors).

# Table of Contents

## Metric Based

Fairness Through Awareness (2012)

Probably Approximately Metric-Fair Learning (2018)

## Bandit Problem Based

Online Learning with an Unknown Fairness Metric (2018)

## Counterfactual Based

Counterfactual Fairness (2017)

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## Introduction

- ▶ We consider the linear contextual bandits problem with strong individual fairness constraints.
- ▶ In this paper, a class of distance functions is specified by Mahalanobis distance (i.e., for some matrix  $A$ ,  $d(x_1, x_2) = \|Ax_1 - Ax_2\|_2$ ).

# Linear Contextual Bandits

## ► Notation

- $t$ : round,  $t \in [T]$
  - $k$ : number of multi-arms
  - $\mathbf{x}_i^t \in \mathbb{R}^d$ : contexts vector of an arm  $i$  in round  $t$
  - $i^t$ : chosen arm at round  $t$  after observing contexts
  - $r_{i^t}^t$ : after choosing an arm, observed some stochastic reward s.t.  $r_{i^t}^t$  is sub-gaussian and  $\mathbb{E}[r_{i^t}^t] = \langle \mathbf{x}_{i^t}^t, \theta \rangle$  where  $\theta \in \mathbb{R}^d$  is a coefficient vector
  - $h^t = ((\mathbf{x}^1, i^1, r^1), \dots, (\mathbf{x}^{t-1}, i^{t-1}, r^{t-1}))$ : a history at round  $t$
  - $\pi^t = \pi^t(h^t, \mathbf{x}^t) \in \Delta[k]$ : the probability distribution over actions that the algorithm plays action  $i$  at round  $t$
- Note that the algorithm does not observe the reward for the actions not chosen.



## Fairness Constraints and Feedback

- ▶ **Def 1.** Algorithm  $L$  is Lipschitz-fair on round  $t$  w.r.t.  $d$  if for all  $i, j$ :

$$|\pi_i^t - \pi_j^t| \leq d(x_i^t, x_j^t).$$

- ▶ **Def 2 (Fairness Oracle).** Given  $d$ , a fairness oracle  $O_d$  defined as follows:

$$O_d(x^t, \pi^t) = \{(i, j) : |\pi_i^t - \pi_j^t| > d(x_i^t, x_j^t)\}$$

- ▶ Assumption: algorithm  $L$  have access to a fairness oracle, use this feedback to learn  $d$

## Best Fair Policy

In round  $t = 1, \dots, T$ ,

1. Parameter estimation:  $\hat{\theta}^t = (\mathbf{X}^{t\top} \mathbf{X}^t + \lambda I)^{-1} \mathbf{X}^{t\top} \mathbf{R}^t$   
where  $\mathbf{X}^t = [x_{i_1}^1, \dots, x_{i_{t-1}}^{t-1}]$  and  $\mathbf{R}^t = [r_{i_1}^1, \dots, r_{i_{t-1}}^{t-1}]$
2. Reward estimation and UCB (upper confidence bound):  $\tilde{r}_i^t = \langle \hat{\theta}^t, x_i^t \rangle$  and  $\hat{r}_i^t = \tilde{r}_i^t + B_i^t$  with  $P(|r_i^t - \tilde{r}_i^t| \leq B_i^t) = 1 - \delta$ .
3. Policy estimation: given  $\hat{\mathbf{r}}^t = (\hat{r}_1^t, \dots, \hat{r}_k^t)$ ,  $\hat{\mathbf{d}}^t = (\hat{d}(x_i^t, x_j^t))_{i < j}$ ,

$$\pi^t(\hat{\mathbf{r}}^t, \hat{\mathbf{d}}^t) = \operatorname{argmax}_{\pi \in \Delta[k]} \sum_{i=1}^k \pi_i \hat{r}_i^t \quad (2)$$

subject to  $|\pi_i - \pi_j| \leq \hat{d}(x_i^t, x_j^t), \forall (i, j)$

## Estimation for $d$

$$\pi^t = \pi(\bar{r}^t, \hat{d}^t)$$

Pull an arm  $i^t$  according to  $\pi^t$  and receive a reward  $r_{i^t}^t$

$$S = \mathbf{O}_d(x^t, \pi^t)$$

$$R = \{(i, j) \mid (i, j) \notin S \wedge |p_i^t - p_j^t| = \hat{d}_{ij}^t\}$$

**for**  $(i, j) \in S$  **do**

    | **DistanceEstimator** $_{ij}$ .*feedback*( $\perp$ )  
    |  $v_{ij}^t = 1$

**end**

**for**  $(i, j) \in R$  **do**

    | **DistanceEstimator** $_{ij}$ .*feedback*( $\top$ )  
    |  $v_{ij}^t = 1$

**end**

# Table of Contents

## Metric Based

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## Multiple classifications based

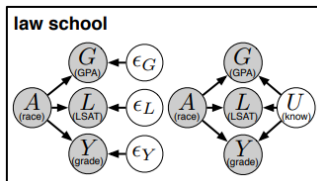
Average Individual Fairness (2019)

## Notation

- ▶  $A$  : the set of protected attributes
- ▶  $X$  : observable attributes
- ▶  $U$  : latent attributes
- ▶  $Y$  : the outcome to be predicted
- ▶  $\hat{Y}$  : predictor, a random variable that depends on  $A, X, U$

## Causal Models and Counterfactuals

- ▶ Causal model is defined by  $(U, V, F)$
- ▶  $V$ : observable variables
- ▶  $U$ : set of latent background variable, which are factors not caused by  $V$
- ▶  $F$  is a set of functions  $\{f_1, \dots, f_n\}$  such that  $V_i = f_i(pa_i, U_{pa_i})$  where  $pa_i \subseteq V \setminus \{V_i\}$ ,  $U_{pa_i} \subseteq U$   
 $pa_i$  refers to the “parents” of  $V_i$



## Causal Models and Counterfactuals

- ▶ Intervention on variable  $V_i$   
substitution of equation  $V_i = f_i(pa_i, U_{pa_i})$  with the equation  $V_i = v$
- ▶ Counterfactual
  - ▶ the value of  $Y$  if  $A$  had taken value  $a$
  - ▶ solution for  $Y$  given  $U = u$  where the equations for  $A$  are replaced with  $A = a$
  - ▶  $Y_{A \leftarrow a}(u)$  or  $Y_a$

## Counterfactual Fairness

- ▶ (Definition) Predictor  $Y$  is counterfactually fair if any context  $X = x$  and  $A = a$

$$P(Y_{A \leftarrow a}(U) = y | X = x, A = a) = P(Y_{A \leftarrow a'}(U) = y | X = x, A = a)$$

for all  $y$  and for any value  $a'$  attainable by  $A$



## Counterfactual Fairness

- ▶ (Lemma) Let  $\mathcal{G}$  be the causal graph of the given model  $(U, V, F)$ . Then  $\hat{Y}$  will be counterfactually fair if it is a function of the non-descendants of  $A$  is invariant with respect to the counterfactual values of  $A$ .

## Algorithm

- ▶  $\hat{Y} \equiv g_\theta(U, X_{\neq A})$  : predictor parameterized by  $\theta$
- ▶  $X_{\neq A} \subset X$  : non-descendants of  $A$
- ▶  $\mathcal{D} \equiv \{(A^{(i)}, X^{(i)}, Y^{(i)}) : i = 1, \dots, n\}$  : training data
- ▶  $l(\cdot, \cdot)$  : loss function (squared loss or log-likelihood)

- 1: **procedure** FAIRLEARNING( $\mathcal{D}, \mathcal{M}$ ) ▷ Learned parameters  $\hat{\theta}$
- 2: For each data point  $i \in \mathcal{D}$ , sample  $m$  MCMC samples  $U_1^{(i)}, \dots, U_m^{(i)} \sim P_{\mathcal{M}}(U \mid x^{(i)}, a^{(i)})$ .
- 3: Let  $\mathcal{D}'$  be the augmented dataset where each point  $(a^{(i)}, x^{(i)}, y^{(i)})$  in  $\mathcal{D}$  is replaced with the corresponding  $m$  points  $\{(a^{(i)}, x^{(i)}, y^{(i)}, u_j^{(i)})\}$ .
- 4:  $\hat{\theta} \leftarrow \operatorname{argmin}_\theta \sum_{i' \in \mathcal{D}'} l(y^{(i')}, g_\theta(U^{(i')}, x_{\neq A}^{(i')}))$ .
- 5: **end procedure**

# Table of Contents

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Online Learning with an Unknown Fairness Metric (2018)

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## Introduction

- ▶ The previous method requires that one provides the causal model that generated the data at hand
- ▶ There are infinitely many structural equations compatible with the same observable distribution.
- ▶ It is desirable to integrate competing causal models to provide counterfactually fair decisions

## Definition

- ▶  $(\epsilon, 0)$ -ACF (Approximate Counterfactual Fairness)

A predictor  $f(\mathcal{X}, A)$  satisfies  $(\epsilon, 0)$ -ACF if given the sensitivity attribute  $A = a$  and any instantiation  $x$  of the other observed variable  $\mathcal{X}$ , we have that

$$|f(x_{A \leftarrow a}, a) - f(x_{A \leftarrow a'}, a')| < \epsilon$$

for all  $a' \neq a$

- ▶  $(\epsilon, \delta)$ -ACF

$f$  satisfies  $(\epsilon, \delta)$ -ACF if

$$\mathbb{P}_U(|f(\mathcal{X}_{A \leftarrow a}, a) - f(\mathcal{X}_{A \leftarrow a'}, a')| < \epsilon | \mathcal{X} = x, A = a) > 1 - \delta$$

## Algorithm

- ▶ objective function :

$$\min_f \frac{1}{n} \sum_{i=1}^n l(f(x_i, a_i), y_i) + \lambda \sum_{j=1}^m \frac{1}{n} \sum_{i=1}^n \sum_{a' \neq a_i} \mu_j(f, x_i, a_i, a') \quad (7)$$

where  $\mu_j(f, x_i, a_i, a') := \mathbb{I}[|f(x_{i,A \leftarrow a_i}, a_i) - f(x_{i,A \leftarrow a'}, a')| > \epsilon]$

- ▶ surrogated version :

$$\mu_j(f, x_i, a_i, a') := \max\{0, |f(x_{i,A \leftarrow a_i}, a_i) - f(x_{i,A \leftarrow a'}, a')| - \epsilon\}$$

# Algorithm

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**Algorithm 1** Multi-World Fairness

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- 1: **Input:** features  $\mathbf{X} = [\mathbf{x}_1, \dots, \mathbf{x}_n]$ , labels  $\mathbf{y} = [y_1, \dots, y_n]$ , sensitive attributes  $\mathbf{a} = [a_1, \dots, a_n]$ , privacy parameters  $(\epsilon, \delta)$ , trade-off parameters  $\mathcal{L} = [\lambda_1, \dots, \lambda_l]$ .
  - 2: **Fit causal models:**  $M_1, \dots, M_m$  using  $\mathbf{X}, \mathbf{a}$  (and possibly  $\mathbf{y}$ ).
  - 3: **Sample counterfactuals:**  $\mathcal{X}_{A^1 \leftarrow a'}, \dots, \mathcal{X}_{A^m \leftarrow a'}$  for all unobserved values  $a'$ .
  - 4: **for**  $\lambda \in \mathcal{L}$  **do**
  - 5:     Initialize classifier  $f_\lambda$ .
  - 6:     **while** loop until convergence **do**
  - 7:         Select random batches  $\mathbf{X}_b$  of inputs and batch of counterfactuals  $\mathbf{X}_{A^1 \leftarrow a'}, \dots, \mathbf{X}_{A^m \leftarrow a'}$ .
  - 8:         Compute the gradient of equation (7).
  - 9:         Update  $f_\lambda$  using any stochastic gradient optimization method.
  - 10:     **end while**
  - 11: **end for**
  - 12: **Select model**  $f_\lambda$ : For deterministic models select the smallest  $\lambda$  such that equation (5) using  $f_\lambda$  holds. For non-deterministic models select the  $\lambda$  that corresponds to  $\delta$  given  $f_\lambda$ .
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## Introduction

- ▶ In this paper, consider multiple classification tasks.  
(ex. ads for internet users, public school admissions)
- ▶ Average Individual Fairness constraints: standard statistics (such as error or FP/FN rates) should be approximately equalized across all individuals
- ▶ Here, 'rate' is defined as the average over classification tasks.
- ▶ Given a sample of individuals and classification problems, authors design an algorithm for the fair empirical risk minimization task.

## Notations

- ▶  $i \in [n]$ : index for an individual,  $j \in [m]$ : index for a classification task
- ▶  $\mathcal{P}$ : probability measure over  $\mathcal{X}$ ,  $\mathcal{Q}$ : probability measure over the space of problems  $\mathcal{F}$
- ▶ Dataset:  $D = \{\mathbf{x}_i, (f_j(x_i))_{j=1}^m\}_{i=1}^n$  where  $f_j(x_i) \in \{0, 1\}$  is the label corresponding to  $\mathbf{x}_i$  for the  $j$ th classification task.
- ▶ Denote  $\mathbf{p} = (p_1, p_2, \dots, p_m)$  as learning  $m$  randomized classifiers, where  $p_j$  is the learned classifier for the  $j$ th classification task.

## Definitions

- ▶ **Def 1.** (Individual and Overall Error Rates)

The individual error rate of  $\mathbf{x}$  incurred by  $\mathbf{p}$  is defined as follows:

$$\mathcal{E}(\mathbf{x}, \mathbf{p}; \mathcal{Q}) = \mathbb{E}_{f \sim \mathcal{Q}} [\mathbb{P}_{h \sim \mathbf{p}_f} [h(\mathbf{x}) \neq f(\mathbf{x})]]$$

The overall error rate of  $\mathbf{p}$  is defined as follows:

$$\text{err}(\mathbf{p}; \mathcal{P}, \mathcal{Q}) = \mathbb{E}_{\mathbf{x} \sim \mathcal{P}} [\mathcal{E}(\mathbf{x}, \mathbf{p}; \mathcal{Q})]$$

- ▶ **Def 2.** (Average Individual Fairness, AIF)

We say  $\mathbf{p}$  satisfies “ $(\alpha, \beta)$ -AIF” w.r.t.  $(\mathcal{P}, \mathcal{Q})$  if there exists  $\gamma \geq 0$  s.t.:

$$\mathbb{P}_{\mathbf{x} \sim \mathcal{P}} (|\mathcal{E}(\mathbf{x}, \mathbf{p}; \mathcal{Q}) - \gamma| > \alpha) \leq \beta$$

- ▶ **Fair Learning Problem subject to  $(\alpha, 0)$ -AIF**

$$\begin{aligned} & \min_{\mathbf{p}, \gamma \in [0, 1]} \text{err}(\mathbf{p}; \mathcal{P}, \mathcal{Q}) \\ & \text{s.t. } \forall \mathbf{x} \in \mathcal{X} : |\mathcal{E}(\mathbf{x}, \mathbf{p}; \mathcal{Q}) - \gamma| \leq \alpha \end{aligned}$$

## Method-Empirical version

- ▶ The empirical versions of the overall error rate and the individual error rates can be expressed as:

$$\text{err}(\mathbf{p}; \hat{\mathcal{P}}, \hat{\mathcal{Q}}) = \frac{1}{n} \sum_{i=1}^n \mathcal{E}(\mathbf{x}_i, \mathbf{p}; \hat{\mathcal{Q}}) = \frac{1}{n} \sum_{i=1}^n \frac{1}{m} \sum_{j=1}^m \mathbb{P}_{h_j \sim p_j} [h_j(\mathbf{x}_i) \neq f_j(\mathbf{x}_i)]$$

- ▶ **Empirical Fair Learning Problem**

$$\begin{aligned} & \min_{\mathbf{p}, \gamma \in [0,1]} \text{err}(\mathbf{p}; \hat{\mathcal{P}}, \hat{\mathcal{Q}}) \\ & \text{s.t. } \forall \mathbf{x} \in \mathcal{X} : |\mathcal{E}(\mathbf{x}, \mathbf{p}; \hat{\mathcal{Q}}) - \gamma| \leq \underbrace{2\alpha}_{\text{slightly relaxed}} \end{aligned}$$