Neural Network Pruning

(literature review)

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Sparse

- Neural network pruning
- By removing unimportant weights from a network, several improvements can be expected.
  - Better generalization error.
  - Reduce hardware or storage requirements without affecting their accuracy.

Neural Network Pruning

- Pruning strategies
- Pre-train(Growing) / Prune / Fine-tune.
  1. Train a large, over-parameterized model.
     - Pre-trained is required/not required.
  2. Calculate the importance measure of each parameters, and prune each parameters based on the calculated measure.
     - Importance measures(magnitudes/Increment of loss,)
     - Structured pruning/Unstructured pruning.
  3. Fine-tune the pruned model.
     - Retain/Reinitialize/Rewind
  4. Repeat 1$\sim$3.
     - Single-shot/Iterative.

Notation

- Parameters: $\boldsymbol{\theta}$
- Initial value: $\boldsymbol{\theta}^0$, pre-trained: $\boldsymbol{\theta}^*$
- q-th parameters: θ_q, parameters with 0 in q-th component: $\boldsymbol{\theta}_{-q}$.
- Neural network $f(x; \boldsymbol{\theta})$
- $\boldsymbol{m}$ indicates masking variables which consists with 0 or 1.
- Pruned network $f(x; \boldsymbol{m} \odot \boldsymbol{\theta})$
- Cross-Entropy: $\mathcal{L}(\boldsymbol{\theta}) = \sum_{i=1}^{n} \ell(y_i, f_\theta(x_i))$
- Hessian (Fisher information matrix)

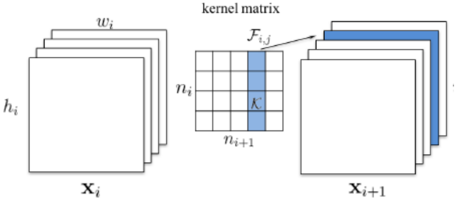
  $\mathbf{H} = \mathbb{E}[\nabla_{\boldsymbol{\theta}} \log p(y|x; \boldsymbol{\theta}) \nabla_{\boldsymbol{\theta}} \log p(y|x; \boldsymbol{\theta})^\top]$

1. Learning Both Weights and Connections for Effiecient Neural Networks.(2015,NIPS)

- Pre-trained/magnitude/unstructured/retain/iterative
- Method:
  1. Pre-train a given architecture.
  2. Prune the connections with their magnitude.($|\theta^*|$)
     (Threshold: $C \times$ standard deviation of the layer's weights)
  3. Fine-tuning with retaining pre-trained values.
     - $f(x; \boldsymbol{m} \odot \boldsymbol{\theta}^*)$
  4. Iteratively implemented.

2. Pruning Filters for Efficient Convnets (2017, ICLR)

- Pre-trained/magnitude/structured/retain/iterative
- Pruning filters in Convolution layer.
- n_i: the number of channels for the ith convolutional layer
- h_i/w_i be the height/widt of the input feature maps.
- $\mathcal{F}_{i,j} \in \mathbb{R}^{n_i \times k \times k}$, $\mathcal{F}_i \in \mathbb{R}^{n_{i+1} \times n_i \times k \times k}$
- When a filter $\mathcal{F}_{i,j}$ is pruned, its corresponding feature map $\boldsymbol{x}_{i+1,j}$ is removed. $n_i k^2 h_{i+1} w_{i+1}$



2. Pruning Filters for Efficient Convnets (2017, ICLR)

- The importance of a filter in each layer is calculating by the sum of its absolute weights.
- We prune same ratio for all layers in the same stage.
- Pruning filter results in pruning the corresponding activation.
  - Independent pruning
  - Greedy pruning(It does not consider the kernels for the previously pruned feature maps while calculating the importance)
- Retraining
  - Prune once and retrain
  - Prune and retrain iteratively
- We prune same ratio for all layers in the same stage.

2. Pruning Filters for Efficient Convnets (2017, ICLR)

- The procedure of pruning m filters from the ith convolutional layer is as follows:
  1. Pre-train a given architecture.
  2. Calculate the importance of jth filter $(\mathcal{F}_{i,j})$ $s_j = \sum_{l=1}^{n_i} \sum |\mathcal{K}_l|$.
  3. Prune m filters with the smallest values and their corresponding feature maps.

3. The Lottery Ticket Hypothesis Finding Sparse, Trainable Neural Network. (2018,ICML)

- Pre-trained/magnitude/unstructured/retain/iterative
- Method:
  1. Pre-train the given architecture
  2. Sort the connections($\boldsymbol{\theta}$) by its magnitudes within layer.
  3. Prune the connections with their magnitude.
     (with certain ratio p)
  4. Fine-tuning by rewinding original initial values.
     - $f(x; \boldsymbol{m} \odot \boldsymbol{\theta_0})$
  5. Iteratively implemented.

3. The Lottery Ticket Hypothesis Finding Sparse, Trainable Neural Network. (2018,ICML)

- There exists the sub-networks with favorable training properties within larger over-parameterized models.
  - Winning tickets $f(x; m \odot \theta_0)$
  - $\|m\|_0 \ll |\theta|$(fewer parameters) commensurate training time and accuracy.
- To find this lucky sub-network, they iteratively prune the lowest magnitude unpruned connections.
- Pruning with rewinding is much better than reinitialization
- Many papers struggle to figure it out.
  - Reinitializing with preserving sign of parameters shows comparable performance
  - Prune weight based on their change in magnitude between initialization and the end of training.
  - Prune weight globally in larger model.

4. Optimal Brain Damage (1990, NIPS)

- Pre-trained/Hessian based method/unstructured/retain/iterative
- The importance of the parameter is measured by the effect on the training loss by enforcing that parameter to be zero.
- From a pre-trained $\boldsymbol{\theta}^*$,

$$\mathbf{I}(\theta_q^*) = \mathcal{L}(\boldsymbol{\theta}^*) - \mathcal{L}(\boldsymbol{\theta}_{-q}^*)$$

- Appoximate the importance(saliency) of θ_q^* by second derivative approiximation.

4. Optimal Brain Damage (1990, NIPS)

- Local maximum $\boldsymbol{\theta}^*$
- Approximate the objective function at $\boldsymbol{\theta}^*$ (Hessian based methods)
- $\Delta\mathcal{L} = \underbrace{\frac{\partial \mathcal{L}}{\partial \boldsymbol{\theta}}^\top \Delta\boldsymbol{\theta}}_{\approx 0} + \frac{1}{2}\Delta\boldsymbol{\theta}^\top \mathbf{H} \Delta\boldsymbol{\theta} + \mathcal{O}\left(\|\Delta\boldsymbol{\theta}\|^3\right)$
- Then, $\mathbf{I}(\theta_q^*)$ can be approximated by $\frac{1}{2}{\theta_q^*}^2\mathbf{H}_{qq}$
- Diagonal assumption on $\mathbf{H}$(Independence assumption.)

4. Optimal Brain Damage (1990, NIPS)

1. Pre-train the given architecture to estimate local maximum $\boldsymbol{\theta}^*$
2. Compute the second derivatives H_{qq} for each parameters
3. Compute the saliences for each parameters: $s_k = \frac{1}{2} H_{qq} {\theta_q^*}^2$
4. Sort the parameters by saliency and prune the paramters
5. Iteratively implement

5. Second Order Derivatives for Network Pruning, Optimal Brain Surgeon (1993, NIPS)

- Pre-trained/Hessian based method/unstructured/retain/single-shot
- No restrictive assumptions on the Hessian.(Dependence assumption)
- Does not require retraining after the pruning of a weight.
- $\Delta\boldsymbol{\theta} \neq \boldsymbol{\theta}^* - \boldsymbol{\theta}^*_{-q}$
- $e_q^\top \Delta\boldsymbol{\theta} + \theta_q = 0$

5. Second Order Derivatives for Network Pruning, Optimal Brain Surgeon (1993, NIPS)

- The importance(saliency) of each weight is calculated by solving the following constrained optimization problem
- $e_q^\top \Delta\theta + \theta_q = 0$

$$\min_{\Delta\boldsymbol{\theta}} \frac{1}{2}\Delta\boldsymbol{\theta}^\top \mathbf{H}\Delta\boldsymbol{\theta} \text{ s.t. } \mathbf{e}_q^\top \Delta\boldsymbol{\theta} + \boldsymbol{\theta}_q^* = 0$$

- Lagrangian form

$$L = \frac{1}{2}\Delta\boldsymbol{\theta}^\top \mathbf{H}\Delta\boldsymbol{\theta} + \lambda(e_q^\top \Delta\theta + \theta_q)$$

5. Second Order Derivatives for Network Pruning, Optimal Brain Surgeon (1993, NIPS)

- By taking functional derivatives,

$$\Delta\boldsymbol{\theta} = -\frac{\boldsymbol{\theta}_q^*}{[\mathbf{H}^{-1}]_{qq}}\mathbf{H}^{-1}\mathbf{e}_q \text{ and } \Delta\mathcal{L}_q = \frac{1}{2}\frac{\left(\boldsymbol{\theta}_q^*\right)^2}{[\mathbf{H}^{-1}]_{qq}}$$

5. Second Order Derivatives for Network Pruning, Optimal Brain Surgeon (1993, NIPS)

- $H \approx \frac{1}{n}\sum_{i=1}^{n} \frac{\partial f_i(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}^{\top} \frac{\partial^2 \mathcal{L}(y_i, f_i(\boldsymbol{\theta}))}{\partial f_i^2} \frac{\partial f_i(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}$
- Denote $\boldsymbol{X}_i = \frac{\partial f_i(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}$, $\boldsymbol{A}_i = \frac{\partial^2 \mathcal{L}(y_i, f_i(\boldsymbol{\theta}))}{\partial f_i^2}$
- with $\boldsymbol{H}_0 = \alpha \boldsymbol{I}$ and $\boldsymbol{H}_n = \boldsymbol{H}$ and $10^{-8} \le \alpha \le 10^{-4}$

$$\boldsymbol{H}_{i+1} := \boldsymbol{H}_i + \frac{1}{n}\boldsymbol{X}_{i+1}^{\top}\boldsymbol{A}_i\boldsymbol{X}_{i+1} \quad i = 0, \ldots, n-1$$

- Recursively,

$$\boldsymbol{H}_{i+1}^{-1} = \boldsymbol{H}_i^{-1} - \boldsymbol{H}_i^{-1}\boldsymbol{X}_{i+1}^{\top}\left(n\boldsymbol{A}_i + \boldsymbol{X}_{i+1}\boldsymbol{H}_i^{-1}\boldsymbol{X}_i^{\top}\right)^{-1}\boldsymbol{X}_{i+1}\boldsymbol{H}_i^{-1}$$

6. EigenDamage: Structured Pruning in the Kronecker-Factored Eigenbasis (2019, ICML)

- Pre-trained/Hessian based method/structured/retain/iterative.
- High computational cost of computing second derivatives.
- Kronecker-factored approximate culvature.(K-FAC)
- Structured prunning
- Present two methods
  - Kron-OBD, Kron-OBS.
  - EigenDamage.

6. EigenDamage: Structured Pruning in the Kronecker-Factored Eigenbasis (2019, ICML)

- Kronecker-Factored Approximation Curvature.
- Kronecker product($mp \times nq$) of $\boldsymbol{M}(m \times n)$, $\boldsymbol{N}(p \times q)$

$$\boldsymbol{M} \otimes \boldsymbol{N} = \begin{pmatrix} \boldsymbol{M}_{11}\boldsymbol{N} & \cdots & \boldsymbol{M}_{1n}\boldsymbol{N} \\ \vdots & & \vdots \\ \boldsymbol{M}_{m1}\boldsymbol{N} & \cdots & \boldsymbol{M}_{mn}\boldsymbol{N} \end{pmatrix}$$

6. EigenDamage: Structured Pruning in the Kronecker-Factored Eigenbasis (2019, ICML)

- Layerwise independence.
- Input activations $a \in \mathbb{R}^n$, weight matrix $W \in \mathbb{R}^{n \times m}$, output $\mathbb{R}^m (s = W^T a)$
- $\nabla_{\boldsymbol{W}} \mathcal{L} = \boldsymbol{a}(\nabla_{\boldsymbol{s}} \mathcal{L})^\top$
- K-FAC decomposes this layer's Fisher matrix.

$$\begin{aligned}
\boldsymbol{H} &= \mathbb{E}[\text{vec}\{\nabla_W \mathcal{L}\}\text{vec}\{\nabla_W \mathcal{L}\}^T] \\
&= \mathbb{E}[\{\nabla_s \mathcal{L}\}\{\nabla_s \mathcal{L}\}^T \otimes aa^T] \\
&\approx \mathbb{E}[\{\nabla_s \mathcal{L}\}\{\nabla_s \mathcal{L}\}^T] \otimes \mathbb{E}[aa^T] = S \otimes A
\end{aligned}$$

6. EigenDamage: Structured Pruning in the Kronecker-Factored Eigenbasis (2019, ICML)

- Efficient computation.($\mathbf{Q}$ and $\mathbf{\Lambda}$)

$$\mathbf{H} \cdot X = \mathbf{S} \otimes \mathbf{A} \cdot \mathbf{X} = \mathbf{A}\mathbf{X}\mathbf{S}^\top$$

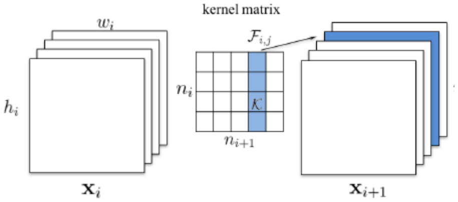
$$\mathbf{H}^{-1} = (\mathbf{S} \otimes \mathbf{A})^{-1} = \mathbf{S}^{-1} \otimes \mathbf{A}^{-1}$$

$$\mathbf{H} = (\mathbf{Q_S} \otimes \mathbf{Q_A})(\mathbf{\Lambda}_S \otimes \mathbf{\Lambda}_A)(\mathbf{Q}_S \otimes \mathbf{Q_A})^\top$$

- $\mathbf{Q_S} \otimes \mathbf{Q_A}$ gives the eigen basis of the Kronecker product.

6. EigenDamage: Structured Pruning in the Kronecker-Factored Eigenbasis (2019, ICML)

- For ith convolution layer,
- n_i: number of channels, h_i: height size, w_i: weight size, k: filter size.
- $\boldsymbol{x}_i \in \mathbb{R}^{n_i \times h_i \times w_i}$, $\boldsymbol{x}_{i+1} \in \mathbb{R}^{n_{i+1} \times h_{i+1} \times w_{i+1}}$, $\mathcal{F}_{i,j} \in \mathbb{R}^{n_i \times k^2}$, $\mathcal{F}_i \in \mathbb{R}^{n_i \times n_{i+1} \times k^2}$



6. EigenDamage: Structured Pruning in the Kronecker-Factored Eigenbasis (2019, ICML)

- $\mathcal{I}_{i+1} = [h_{i+1}] \times [w_{i+1}]$, spatial location index for output channel.
- $\ell \in \mathcal{I}_{i+1}$, $\boldsymbol{x}_{i+1}^{(\ell)} \in \mathbb{R}^{n_{i+1}}$
- Like in the fully connected layer case, $\Delta_{\boldsymbol{W}} \mathcal{L} = \boldsymbol{a}(\Delta_{\boldsymbol{s}} \mathcal{L})^{\top}$
  the gradient of the kernel matrix $\mathcal{F}_i = \sum_{\ell \in \mathcal{I}_{i+1}} x_i^{p(\ell)} \nabla_{x_{i+1}^{(\ell)}} \mathcal{L}^{\top}$
- $\boldsymbol{x}_i^{p(\ell)} \in \mathbb{R}^{n_i k^2}$ is corresponding patch for l-th spatial location.
- $\nabla_{\mathcal{F}_{i,j}} \mathcal{L} = \sum_{l \in \mathcal{I}_{i+1}} \boldsymbol{x}_i^{p(\ell)} \nabla_{\boldsymbol{x}_{i+1}^{(\ell)}} \mathcal{L}^{T}, j = 1, \ldots, n_{i+1}$

6. EigenDamage: Structured Pruning in the Kronecker-Factored Eigenbasis (2019, ICML)

- Then, Hessian matrix can be computed.

$$
\begin{aligned}
\boldsymbol{H}_i &\approx \sum_{l \in \mathcal{I}_{i+1}} \mathbb{E}[\{\nabla_{x_{i+1}^{(\ell)}} \mathcal{L}\}\{\nabla_{x_{i+1}^{(\ell)}} \mathcal{L}\}^T] \otimes \mathbb{E}[x_i^{p(\ell)} x_i^{p(\ell)\,T}] \\
&\approx \left(\frac{1}{|\mathcal{I}_{i+1}|} \sum \mathbb{E}[\{\nabla_{x_{i+1}^{(\ell)}} \mathcal{L}\}\{\nabla_{x_{i+1}^{(\ell)}} \mathcal{L}\}^T]\right) \otimes \left(\sum_{\ell \in \mathcal{I}_{i+1}} \mathbb{E}[x_i^{p(\ell)} x_i^{p(\ell)\,T}]\right) \\
&:= \boldsymbol{S} \otimes \boldsymbol{A}
\end{aligned}
$$

where $\boldsymbol{S}$ is $n_{i+1} \times n_{i+1}$ and $\boldsymbol{A}$ is $n_i k^2 \times n_i k^2$ matrix.

6. EigenDamage: Structured Pruning in the Kronecker-Factored Eigenbasis (2019, ICML)

- Without considering the interaction between filters, we can compute the importance of each filter in ith convolution layer

$$\Delta \mathcal{L}_j = \frac{1}{2} \mathcal{F}_{ij}^{*\top} H_{i,j} \mathcal{F}_{ij}^{*}$$

where $\mathcal{F}_{ij}^{*} \in \mathbb{R}^{n_i k^2}$ and $H_{i,j} \in \mathbb{R}^{n_i k_i^2 k^2}$

- (Convolution) layerwise independence.
- Kron-OBD

$$\Delta \mathcal{F}_{ij} = -\mathcal{F}_{ij}^{*} \text{ and } \Delta \mathcal{L}_j = \frac{1}{2} \boldsymbol{S}_{jj} \mathcal{F}_{ij}^{*\top} \boldsymbol{A} \mathcal{F}_{ij}^{*}$$

- Kron-OBS

$$\Delta \mathcal{F}_{ij} = -\frac{\boldsymbol{S}^{-1} \boldsymbol{e}_j \otimes \mathcal{F}_{ij}^{*}}{[\boldsymbol{S}^{-1}]_{jj}} \text{ and } \Delta \mathcal{L}_j = \frac{1}{2} \frac{\mathcal{F}_i^{*\top} \boldsymbol{A} \mathcal{F}_{ij}^{*}}{[\boldsymbol{S}^{-1}]_{jj}}$$

where $\boldsymbol{e}_j$ is the selecting vector with 1 for elements of $\mathcal{F}_{i,j}$

6. EigenDamage: Structured Pruning in the Kronecker-Factored Eigenbasis (2019, ICML)

1. Compute Kronecker factors $\boldsymbol{A}$ and $\boldsymbol{S}$
2. Compute the importance for each filters in each layers.
3. Compute p_{th} percentile of $\Delta\mathcal{L}$ as τ
4. Update $\boldsymbol{\theta}$ as $\Delta\boldsymbol{\theta}$(Kron-OBD or Kron-OBS)
5. Finetune the network.

6. EigenDamage: Structured Pruning in the Kronecker-Factored Eigenbasis (2019, ICML)

- Eigendamage based on $\mathbf{Q_S} \otimes \mathbf{Q_A}$ the eigen basis of the Kronecker product.
- Introduce a novel network reparameterization.

$$\mathcal{F}_i = (\boldsymbol{Q_S} \otimes \boldsymbol{Q_A})\mathcal{F}_i^{'} = \boldsymbol{Q_A}\mathcal{F}_i^{'}\boldsymbol{Q_S}^{\top}, \quad \mathcal{F}_i^{'} = (\boldsymbol{Q_S} \otimes \boldsymbol{Q_A})^{\top}\mathcal{F}_i$$

6. EigenDamage: Structured Pruning in the Kronecker-Factored Eigenbasis (2019, ICML)

1. Compute Kronecker factors $\boldsymbol{A}$ and $\boldsymbol{S}$
2. $Q_S, \boldsymbol{\Lambda}_S = \text{Eigen}(S)$ and $Q_A, \boldsymbol{\Lambda}_A = \text{Eigen}(A)$
3. $\boldsymbol{\Theta} = \mathcal{F}_i^{'} \cdot \text{diag}(\boldsymbol{\Lambda}_A)\,\text{diag}(\boldsymbol{\Lambda}_S) \cdot \mathcal{F}_i^{'}$
4. for all row (or column) in $\boldsymbol{\Theta}$

$$\Delta\mathcal{L}_r = \boldsymbol{\theta}_{r,.}1 \text{ or } (\Delta\mathcal{L}_c = \mathbf{1}^\top \boldsymbol{\theta}_{.,c})$$

5. Remove r_{th} row(or c_{th} column) in $\boldsymbol{W}^{'}$ and r_{th}(or c_{th}) eigenbasis in $\boldsymbol{Q}_A$(or $\boldsymbol{Q}_S$ if $\Delta\mathcal{L}_r$(or $\Delta\mathcal{L}_c \leq \tau$).
6. Compute p_{th} percentile of $\Delta\mathcal{L}$ as τ
7. Finetune the network.

7. SNIP, Single-shot Pruning Based on Connection Sensitivity. (2018, ICML)

- ~~*Pre − trained*~~/Connection sensitivity/unstructured/retain
- Prunes a given network once at initialization prior to training.
- After pruning, the sparse network is trained in the standard way.
- Determines important connections with a mini-batch of data at single shot.
- The importance(Connection sensitivity) of each parameters is calculated as follow.

$$S(\theta_q) = \lim_{\epsilon \to 0} \left| \frac{\mathcal{L}(\theta_0) - \mathcal{L}(\theta_0 + \epsilon \delta_q)}{\epsilon} \right| = \left| \theta_q \frac{\partial \mathcal{L}}{\partial \theta_q} \right|$$

7. SNIP, Single-shot Pruning Based on Connection Sensitivity. (2018, ICML)

1. Variance scaling initialization
2. Using a mini-batch, Compute the connection sensitivity.

$$S(\theta_q) = \lim_{\epsilon \to 0} \left| \frac{\mathcal{L}(\theta_0) - \mathcal{L}(\theta_0 + \epsilon \delta_q)}{\epsilon} \right| = \left| \theta_q \frac{\partial \mathcal{L}}{\partial \theta_q} \right|$$

3. Sort the parameters by their connection sensitivity and prune the parameters
4. Training the pruned model.

8. Picking Winning Tickets Before Training by Preserving Gradient Flow. (2020, ICML)

- GraSP
- ~~*Pre − trained*~~/Preserving Gradient Flow/unstructured/retain
- It is important to preserve the training dynamics than the loss itself.
- A larger gradient norm indicates that, each gradient update achieves a greater loss reduction,

$$\Delta\mathcal{L}(\theta) = \lim_{\epsilon \to 0} \frac{\mathcal{L}(\theta + \epsilon\nabla\mathcal{L}(\theta)) - \mathcal{L}(\theta)}{\epsilon} = \nabla\mathcal{L}(\theta)^{\top}\nabla\mathcal{L}(\theta)$$

8. Picking Winning Tickets Before Training by Preserving Gradient Flow. (2020, ICML)

- Prune the weights whose removal will not reduce the gradient flow.

$$\mathbf{S}(\delta) = \Delta\mathcal{L}\left(\theta_0 + \delta\right) - \underbrace{\Delta\mathcal{L}\left(\boldsymbol{\theta}_0\right)}_{\text{Const}} = 2\delta^\top \nabla^2 \mathcal{L}\left(\theta_0\right) \nabla \mathcal{L}\left(\theta_0\right) + \mathcal{O}\left(\|\delta\|_2^2\right)$$

$$= 2\delta^\top \mathbf{H}\mathbf{g} + \mathcal{O}\left(\|\delta\|_2^2\right)$$

- The importance of weights,

$$\mathbf{S}(-\boldsymbol{\theta}) = -\boldsymbol{\theta} \odot \mathbf{H}\mathbf{g}$$

  - The larger the score of a weight the lower its importance.
  - $\mathbf{H} = \mathbf{I}$, equals to SNIP.

8 .Picking Winning Tickets Before Training by Preserving Gradient Flow. (2020, ICML)

1. $\mathcal{D}_b = \{(\mathrm{x}_i, \mathrm{y}_i)\}_{i=1}^{b} \sim \mathcal{D}$
2. Compute the Hessian-gradient product Hg
3. $\mathrm{S}(-\theta_0) = -\theta_0 \odot \mathrm{Hg}$
4. Compute p_{th} percentile of $\mathrm{S}(-\theta_0)$ as τ
5. $\mathbf{m} = \mathbf{S}(-\boldsymbol{\theta}_0) < \tau$
6. Train the network $f_{\mathrm{m}\odot\theta}$ on $\mathcal{D}$ until convergence.

9. A Signal Propagation Perspective for Pruning Neural Networks at Initialization. (2020, arXiv)

- Faithful connection sensitivity through layerwise dynamical isometry.
- Layerwise Jacobian

$$\mathbf{J}^{l-1,l} = \frac{\partial \mathbf{x}^l}{\partial \mathbf{x}^{l-1}} = \mathbf{D}^l \mathbf{W}^l$$

- Layerwise dynamical isometry is that Layerwise Jacobian's singular values are exactly 1.
- Reinitialize the weights after pruining by

$$\min_{\mathbf{W}^l} \left\| \left(\mathbf{m}^l \odot \mathbf{W}^l\right)^T \left(\mathbf{m}^l \odot \mathbf{W}^l\right) - \mathbf{I}^l \right\|_F$$

The end

The end.