## Neural Basis Models for Interpretability (NeurIPS, 2022)

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Given a input  $\boldsymbol{x} = (x_1, \dots, x_D) \in \mathbb{R}^D$ , a label  $y \in \mathbb{R}$ , a link function  $g : \mathbb{R}^D \to \mathbb{R}$ ,  $g(\boldsymbol{x})$  can be expressed as

$$GAM : g(x) = f_0 + \sum_{i=1}^{D} f_i(x_i)$$
$$GA^2M : g(x) = f_0 + \sum_{i=1}^{D} f_i(x_i) + \sum_{i=1}^{D} \sum_{j>i} f_{ij}(x_i, x_j)$$

for some bias  $f_0 \in \mathbb{R}$ , univariate functions  $f_i$ , and bivariate functions  $f_{ij} : \mathbb{R} \to \mathbb{R}$ .

$$\mathsf{GAM}: g(\boldsymbol{x}) = f_0 + \sum_{i=1}^{D} f_i(x_i)$$

Neural Additive Model (NAM): each  $f_i$  is parametrized by DNN.

Neural Basis Model (NBM): each  $f_i$  is represented as  $f_i(x_i) = \sum_{k=1}^{B} h_k(x_i)a_{ik}$ . And basis functions  $(h_1, \ldots, h_B) : \mathbb{R} \to \mathbb{R}^B$ are parametrized by DNN.

Multiclass GAM : 
$$g_{l}(\boldsymbol{x}) = f_{0l} + \sum_{i=1}^{D} f_{i}(x_{i}) \boldsymbol{w}_{il}$$

Neural Basis Model (NBM): each  $f_i$  is represented as  $f_i(x_i) = \sum_{k=1}^{B} h_k(x_i)a_{ik}$ . And basis functions  $(h_1, \ldots, h_B) : \mathbb{R} \to \mathbb{R}^B$ are parametrized by DNN.

## NBM Extension(NB<sup>2</sup>M)

$$GA^{2}M: g(\boldsymbol{x}) = f_{0} + \sum_{i=1}^{D} f_{i}(x_{i}) + \sum_{i=1}^{D} \sum_{j>i} f_{ij}(x_{i}, x_{j})$$

NB<sup>2</sup>M: each  $f_{ij}$  is represented as  $f_{ij}(x_i, x_j) = \sum_{k=1}^{B} u_k(x_i, x_j) b_{ijk}$ . And additional basis functions  $(u_1, \ldots, u_B) : \mathbb{R}^2 \to \mathbb{R}^B$  are parametrized by DNN.

Extension to multi-class setting can be done in the similar way as for NBM.

If all  $f_i$ s are in an RKHS, then risk converges to 0 as  $n \rightarrow 0$ .  $\Rightarrow B = O(\log D)$  bases are sufficient. The proof seems a little awkward to me.

Rather than tuning this hyperparameter, they recommend setting B = 100 for NBM and B = 200 for NB<sup>2</sup>M as it performs well across a large variety of datasets they experimented with.

(1) Number of parameters : Number of weight parameters needed to learn the model. When the input dimension is large, NBM has far fewer parameters than NAM.

(2) Throughput : The number of data instances processed per second, which directly affects the training speed. NBM are much more efficient than NAM.

(3) Performance : NBM outperform NAM and NODE-GAM(state of the art) on most datasets.

(4) Stability : the functions  $f_i$  of NBM are much more stable than those of NAM.

## (1) Number of parameters and (2) Throughput

| Model                                  | CA Housing   |                  | FICO         |                  | CoverType   |                 | Newsgroups  |               | iNat. Birds  |                |
|--|--------------|------------------|--------------|------------------|-------------|-----------------|-------------|---------------|--------------|----------------|
|  | #par.        | x/sec            | #par.        | x/sec            | #par.       | x/sec           | #par.       | x/sec         | #par.        | <i>x</i> /sec  |
| NAM<br>NBM                             | 54K<br>65K   | 0.5M<br>3.4M×6.8 | 262K<br>68K  | 123K<br>821K×6.7 | 363K<br>70K | 80K<br>530K×6.6 | 984M<br>18M | 23<br>†9K×391 | 2.3M<br>0.5M | 15K<br>74K×4.9 |
| NA <sup>2</sup> M<br>NB <sup>2</sup> M | 243K<br>161K | 119K<br>641K×5.4 | 5.3M<br>0.3M | 6K<br>30K×5.0    | 10M<br>0.5M | 3K<br>15K×5.0   | _           | -             | 320M<br>66M  | 99<br>374×3.8  |

When the input dimension is large, NBM has far fewer parameters than NAM.

NBM are much more efficient than NAM.

## (3) Performance

| Model                     | MIMIC-II                                 | Credit                                   | Click                                    | Epsilon                                  | Higgs                                    | Microsoft                                | Yahoo<br>MSE↓                            | Year<br>MSE↓                          |
|---------------------------|--|--|--|--|--|--|--|---------------------------------------|
|                           | AUROC↑                                   | AUROC↑                                   | Error↓                                   | Error ↓                                  | Error↓                                   | MSE ↓                                    |  |                                       |
| NAM                       | 0.8539<br>±0.0004                        | 0.9766<br>±0.0027                        | 0.3317<br>±0.0005                        | 0.1079<br>±0.0002                        | 0.2972<br>±0.0001                        | 0.5824<br>±0.0002                        | 0.6093<br>±0.0003                        | 85.25<br>±0.01                        |
| NODE<br>GAM               | $\underset{\pm 0.0110}{\textbf{0.8320}}$ | $\underset{\pm 0.0110}{\textbf{0.9810}}$ | $\underset{\pm 0.0001}{\textbf{0.3342}}$ | $\underset{\pm 0.0003}{\textbf{0.1040}}$ | $\underset{\pm 0.0001}{\textbf{0.2970}}$ | $\underset{\pm 0.0004}{\textbf{0.5821}}$ | $\underset{\pm 0.0006}{\textbf{0.6101}}$ | <b>85.09</b><br>±0.01                 |
| NBM                       | 0.8549<br>±0.0004                        | <b>0.9829</b><br>±0.0014                 | $\underset{\pm 0.0002}{\textbf{0.3312}}$ | 0.1038<br>±0.0002                        | <b>0.2969</b><br>±0.0001                 | 0.5817<br>±0.0001                        | 0.6084<br>±0.0001                        | $\underset{\pm 0.01}{\textbf{85.10}}$ |
| NA <sup>2</sup> M         | 0.8639<br>±0.0011                        | 0.9824<br>±0.0032                        | 0.3290<br>±0.0005                        | _  | 0.2555<br>±0.0003                        | 0.5622<br>±0.0003                        | _  | 79.80<br>±0.05                        |
| NODE<br>GA <sup>2</sup> M | 0.8460<br>±0.0110                        | 0.9860<br>±0.0100                        | 0.3307<br>±0.0001                        | $\underset{\pm 0.0002}{\textbf{0.1050}}$ | 0.2566<br>±0.0003                        | 0.5618<br>±0.0003                        | 0.5807<br>±0.0004                        | <b>79.57</b><br>±0.12                 |
| NB <sup>2</sup> M         | 0.8690<br>±0.0010                        | 0.9856<br>±0.0017                        | 0.3286<br>±0.0002                        | _  | 0.2545<br>±0.0002                        | 0.5618<br>±0.0002                        | _  | <b>79.01</b><br>±0.03                 |

NBM outperform NAM and NODE-GAM(SOTA) on most datasets.

(4) Stability



The functions  $f_i$  of NBM are much more stable than those of NAM.